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Naval Research Laboratory
Washington, DC 20375-5000

NRL Memorandum Report 6542

Optical Guiding by Plasma Waves in the Plasma Beat Wave Accelerator

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October 25, 1989

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved OMB No 0704-0188

1a REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b RESTRICTIVE MARKINGS
2a SECURITY CLASSIFICATION AUTHORITY	3 DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
2b DECLASSIFICATION/DOWNGRADING SCHEDULE	
4 PERFORMING ORGANIZATION REPORT NUMBER(S) NRL Memorandum Report 6542	5 MONITORING ORGANIZATION REPORT NUMBER(S)
6a NAME OF PERFORMING ORGANIZATION Naval Research Laboratory 6b OFFICE SYMBOL (If applicable) Code 4790	7a NAME OF MONITORING ORGANIZATION
6c ADDRESS (City, State, and ZIP Code) Washington, DC 20375-5000	7b ADDRESS (City, State, and ZIP Code)
8a NAME OF FUNDING/SPONSORING ORGANIZATION Department of Energy 8b OFFICE SYMBOL (If applicable)	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c ADDRESS (City, State, and ZIP Code) Washington, DC 20545	10 SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO: — PROJECT NO: DOE #AI05-83ER 40117 TASK NO: — WORK UNIT ACCESSION NO: —

11 TITLE (Include Security Classification)

Optical Guiding by Plasma Waves in the Plasma Beat Wave Accelerator

12 PERSONAL AUTHOR(S)

Esarey, E. and Ting, A.

13a TYPE OF REPORT	13b TIME COVERED	14 DATE OF REPORT (Year, Month, Day)	15 PAGE COUNT
Interim	FROM ______ TO ______	1989 October 25	26

16 SUPPLEMENTARY NOTATION

17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Plasma beat wave accelerator Optical guiding Laser plasma interactions

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

As the resonantly driven plasma wave grows to sufficiently large amplitudes, it strongly affects the diffractive properties of the radiation beams in the plasma beat wave accelerator (PBWA). In particular, a large amplitude plasma wave (with phase velocity approximately the speed of light) will break up an initially uniform radiation beam into periodic beamlet segments, of length less than or equal to half a plasma wavelength, which remain optically guided as they propagate. In the PBWA, for an optimal choice of the mismatch between the radiation beat frequency and the ambient plasma frequency, the resonantly driven plasma wave may lead to enhanced focusing of the radiation beams.

20 DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	21 ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a NAME OF RESPONSIBLE INDIVIDUAL E. Esarey	22b TELEPHONE (Include Area Code) (202) 767-7720 22c OFFICE SYMBOL Code 4790

CONTENTS

OPTICAL GUIDING BY PLASMA WAVES IN THE PLASMA BEAT WAVE ACCELERATOR

I. Introduction

Recently much interest has arisen in plasma based accelerator schemes, such as the plasma beat wave accelerator[1] (PBWA), for producing ultra-high energy electrons. This has led to a renewed interest in the study of the propagation of intense radiation beams through a plasma.[2-7] In the PBWA two colinear radiation beams of frequencies ω_1, ω_2 are incident on a uniform plasma. By appropriately choosing the difference in the laser frequencies to be approximately the electron plasma frequency ω_p, $\Delta\omega = \omega_1 - \omega_2 \simeq \omega_p$, where $\omega_p^2/\omega_1^2 << 1$, it is possible for the radiation beat wave to resonantly drive large amplitude electron plasma waves. In the ideal wave breaking limit,[8] the maximum accelerating electric field is given by $E_{wb} = (mc/e)\omega_p \simeq \sqrt{n}$ eV/cm where n is the plasma density in cm^{-3}. For example, $n = 10^{16}$ cm^{-3} gives $E_{wb} \simeq 100$ MeV/cm which implies that an electron, under ideal conditions, may be accelerated to 1 TeV in 100 meters.

To realize such an acceleration scheme it is necessary that the radiation beams propagate at high intensity over distances large compared to the Rayleigh length $z_R = \omega r_s^2/2c$, where r_s is the radiation spot size. In vacuum, radiation diffracts over distances on the order of z_R, which can be relatively short. Hence, in order to maintain high intensity beams it is necessary to rely on focusing enhancement (optical guiding) from the plasma.

Previously, the effects of relativistic self-focusing[2,3] on the radiation beams in the PBWA were examined.[4] Specifically, it was shown that relativistic effects may lead to optical guiding of the radiation beams provide the power in one or both of the beams approached or exceeded the critical power[3] $P_{cr} \simeq 17(\omega/\omega_p)^2$ GW. The previous work, however, neglected the effects of the resonantly driven plasma wave and, hence, the results may only be applied to the leading edge of the radiation beams in which the plasma wave is sufficiently small. For the regions of the radiation beams in which there exists a sufficiently large plasma wave, the effects of this plasma wave on the diffractive properties of the radiation beams cannot be neglected. It has bee[illegible]erved in experiments (Joshi et al.[5]) and in simulations (Mori et al.[6]) that the presen[illegible] the resonantly driven plasma wave can result in enhanced focusing of the radiation beams. This problem was also addressed in the numerical work of Gibbon and Bell,[7] and their results wil be discussed more fully in the following sections.

The present work concerns the effects of the resonantly driven plasma wave on the diffractive properties of the radiation beams in the PBWA. Specifically, a theoretical model is developed which describes under what conditions the plasma wave may lead to enhanced

Manuscript approved August 31 1989

focusing of the radiation beams in the PBWA. This model assumes that the power in the radiation beams is sufficiently far below the critical power P_{cr} so that the effects of relativistic self-focusing may be neglected. It is shown below that a plasma wave of sufficiently large amplitude (with phase velocity $v_{ph} \simeq c$) will tend to break up a radiation beam into periodic axial "beamlet" sections (of period λ_p) in which sections of length $\lesssim \lambda_p/2$ remain optically guided whereas the remainder of the radiation beam continually diffracts. The optically guided beamlet sections occur at the minima in electron density of the plasma wave. Provided the mismatch between the radiation beat frequency and the ambient plasma frequency corresponds to an optimal choice, the plasma wave may lead to focusing enhancement of the radiation beams in the PBWA.

II. Optical Guiding in an Externally Generated Plasma Wave

The physical mechanism for producing optical guiding using a plasma wave (with $v_{ph} \simeq c$) is similar to the mechanism for producing optical guiding using a density channel. Heuristically, this may be understood by considering the index of refraction η of a radiation beam in a plasma with spatial density variations. Refractive optical guiding is the result of modifying the radial profile of the index of refraction $\eta(r)$ such that it exhibits a maximum on axis $\partial\eta/\partial r < 0$. Refractive guiding of a radiation beam along axis becomes possible provided $\partial n/\partial r < 0$. Neglecting the relativistic effects, the index of refraction for a radiation beam of frequency ω is given by $\eta(r) \simeq 1 - (\omega_{p0}/\omega)^2 n(r)/(2n_0)$, where ω_{p0} is the electron plasma frequency in the ambient plasma density n_0 and where $(\omega_{p0}/\omega)^2 << 1$ has been assumed. Hence, an electron density profile $n(r)$ which exhibits a minimum on axis leads to $\partial\eta/\partial r < 0$. In such a way a density channel may be used to optically guide a radiation beam.

In order to help determine how the plasma wave in the PBWA affects the optical guiding of the radiation beams, it is instructive to consider a simpler model problem which consists of a single radiation beam propagating in the presence of an externally generated plasma wave. For simplicity, the density variation δn of the plasma wave is assumed to have a Gaussian radial profile and a phase velocity equal to the speed of light, $\delta n(r, z, t) = \delta n_0 \exp(-r^2/r_p^2) \sin k_{p0}(z - ct)$, where r_p represents the radius of the plasma wave, $k_{p0} = \omega_{p0}/c$ and $\delta n_0 > 0$. Physically, the qualitative diffractive properties may be understood by considering the index of refraction, which for this case is given by

$$\eta \simeq 1 - (\omega_{p0}^2/2\omega^2) \left[1 + (\delta n_0/n_0) \exp\left(-r^2/r_p^2\right) \sin k_{p0}(z - ct)\right] . \qquad (1)$$

Provided δn_0 is sufficiently large, one expects the radiation beam to be focused ($\partial\eta/\partial r < 0$)

over regions where $\sin k_{p0}(z - ct) < 0$ (which corresponds to decreases in the electron density) and defocused ($\partial\eta/\partial r > 0$) in regions where $\sin k_{p0}(z - ct) > 0$ (which corresponds to increases in the plasma density).

To determine how large δn_0 needs to be in order to achieve optical guiding in regions where $\sin k_{p0}(z - ct) < 0$, an evolution equation for the spot size $r_s(z, t)$ of the radiation beam must be derived. This is done by applying the source dependent expansion[9] (SDE) to the reduced wave equation for the radiation field, as was done previously for the case of relativistic optical guiding.[4] For the present case, however, relativistic effects are ignored and the perpendicular current appearing in the wave equation is given by $J_\perp \simeq -ev_q[n_0 + \delta n(r, z, t)]$, where $v_q = ca_\perp$ is the nonrelativistic electron quiver velocity in the normalized vector potential of the radiation field $a_\perp = eA_\perp/(mc^2)$. Introducing the independent variables $\zeta = z - ct$ and $\tau = t$, and assuming that the radiation field is adequately described by the lowest order Gaussian mode of the SDE expansion, $|a_\perp| \simeq (a_0 r_{s0}/r_s)\exp(-r^2/r_s^2)$, then the evolution equation for the radiation spot size is

$$\frac{\partial^2}{\partial\tau^2} r_s(\zeta, \tau) = \frac{4c^4}{\omega^2 r_s^3}\left[1 + k_{p0}^2 r_p^2 \frac{\delta n_0}{n_0} \frac{\sin k_{p0}\zeta}{(1 + 2r_p^2/r_s^2)^2}\right]. \tag{2}$$

The first term on the right of the above equation represents vacuum diffraction whereas the second term represents the diffractive properties of the plasma wave.

The above equation indicates that optical guiding becomes possible, in regions centered around $\zeta = -\pi/2 \pm 2j\pi$ (where j is an integer), provided

$$\frac{\delta n_0}{n_0} \geq \left(\frac{1 + \epsilon^2}{2\pi}\right)^2 \frac{\lambda_{p0}^2}{r_p^2}, \tag{3}$$

where $\epsilon^2 = 2r_p^2/r_s^2$ and $\lambda_{p0} = 2\pi c/\omega_{p0}$ is the ambient plasma wavelength. Once the above inequality is satisfied, focusing occurs in regions centered about $\zeta = -\pi/2 \pm 2j\pi$. This focusing continues until the spot size r_s decreases to the point where ϵ becomes sufficiently large such that the above inequality is no longer satisfied, thus leading to diffraction. In such a way the spot size r_s (in regions centered about $\zeta = -\pi/2 \pm 2j\pi$) oscillates about some matched beam radius and, hence, the radiation is guided. This matched beam radius r_{sm} is given locally by

$$r_{sm}^2(\zeta) = 2r_p^2\left\{k_{p0} r_p [(-\sin k_{p0}\zeta)\delta n_0/n_0]^{1/2} - 1\right\}^{-1}. \tag{4}$$

Notice in regions where $\sin k_{p0}\zeta > 0$, the radiation beam diffracts at a rate greater than vacuum Rayleigh diffraction. It should be pointed out that Eq. (3) also indicates the depth required for a Gaussian density channel to provide optical guiding of a radiation beam. Likewise, Eq. (4) gives the matched beam radius for a Gaussian density channel when $\zeta = -\pi/2$.

This model problem illustrates how a plasma wave of sufficiently large amplitude (such that $\delta n/n_0$ satisfies the above inequality) breaks up an initially uniform radiation beam into "beamlets" centered about $z - ct = -\pi/2 \pm 2j\pi$ which remain optically guided as they propagate. In the limit $(\delta n_0/n_0)k_{p0}^2 r_p^2/(1+\epsilon^2)^2 >> 1$, the optically guided beamlets occur in the regions $\sin k_{p0}\zeta < 0$, whereas the regions $\sin k_{p0}\zeta > 0$ continually diffract. This behavior is illustrated schematically in Fig. 1.

By performing appropriate averages in ζ over a plasma wavelength, it is possible to define global properties of the radiation envelope. For example, performing a power weighted average of $r_s(\zeta)$ over a plasma wavelength defines the effective global spot size $\bar{r}_s$ for the radiation beam. For the case illustrated in Fig. 1, the global spot size $\bar{r}_s$ remains optically guided even though the actual beam envelope r_s contains periodic regions of width $\lambda_p/2$ which continually diffract. Such global descriptions, which result from averaging over a plasma wavelength, remove the beamlet structure of the radiation beam. This removal of the beamlet structure through averaging is important in the following discussions.

III. Optical Guiding in a Resonantly Generated Plasma Wave

In an actual PBWA, optical guiding of the radiation beams is more complicated since the plasma wave is directly produced by the radiation beams. Roughly speaking, the density oscillation of the plasma wave in the PBWA is of the form $\delta n(r,\zeta,\tau) = \delta\hat{n}(r,\zeta,\tau)\sin(\Delta\phi+\theta)$, where $\Delta\phi$ is the beat phase of the two radiation beams $\Delta\phi = \phi_1 - \phi_2$, where $\phi_{1,2} = k_{1,2}z - \omega_{1,2}t$ is the phase of one of the beams (k is the wavenumber) and where θ is the shift in the phase of the plasma wave away from the radiation beat phase. In the small amplitude limit, initially $\delta\hat{n} \sim |a_1||a_2|\zeta$ (where ζ is a measure of the distance from the head of the radiation beams) and $\theta \simeq 0$. As the plasma wave amplitude grows, however, relativistic effects associated with the axial motion of the plasma electrons cause θ to monotonically increase. Saturation occurs when $\theta = \pi/2$, the point at which the plasma wave is 90° out of phase with the radiation beat wave and is no longer resonantly amplified. By introducing a small frequency mismatch[10] $\Delta\omega_0$ into the radiation beams such that $\Delta\omega = \omega_{p0} + \Delta\omega_0$, the point at which $\theta = \pi/2$ may be extended (to a point further behind the head of the radiation beams) which results in a larger amplitude of the saturated plasma wave. An optimal choice[10] of $\Delta\omega_0 = \Delta\omega_0^{opt}$ leads to the maximum value for the plasma wave electric field at saturation, $E_z = E_{sat}^{max}$. In fact, it is possible to show using 1-D nonlinear fluid theory, that the plasma wave phase shift θ becomes negative for small negative values of $\Delta\omega_0$. In particular, when $\Delta\omega_0 = \Delta\omega_0^{opt}$, then initially $E_z = 0$ and $\theta = 0$. As the plasma wave grows, θ decreases to the point where $\theta = -\pi/2$ at which $E_z = E_{sat}^{max}/2$. Beyond this point, θ increases until saturation is reached where $\theta = \pi/2$ and $E_z = E_{sat}^{max}$. This behavior is to be contrasted with the case $\Delta\omega_0 = 0$, in which θ increases monotonically from zero ($E_z = 0$) to $\pi/2$ ($E_z = E_{sat}$).

This behavior may be understood more readily by considering the 1-D nonlinear, cold fluid equations describing the evolution of the plasma wave $\delta n = \delta\hat{n}\sin(\Delta\phi + \theta)$, where $\Delta\phi = \Delta kz + \Delta\omega t$ and $\Delta\omega = \omega_{p0} + \Delta\omega_0$. In the limits $(\Delta\omega_0/\omega_{p0})^2 << 1$, $|a_1|^2 << 1$ and $|a_2|^2 << 1$, the normalized electric field amplitude $\hat{E}_z = eE_z/(mc\omega_{p0}) = \delta\hat{n}/n_0$ and the phase shift θ of the plasma wave obey the evolution equations:[10,11]

$$\frac{d}{dt}\hat{E}_z = \frac{1}{4}\omega_{p0}a_1a_2\cos\theta, \tag{5a}$$

$$\frac{d}{dt}\theta = \frac{1}{2}\omega_{p0}\left(\frac{\Delta\omega_0}{\omega_{p0}} + \frac{9}{32}\hat{E}_z^2\right), \tag{5b}$$

along with the constant of motion

$$\hat{E}_z^3 + \frac{32}{3}\frac{\Delta\omega_0}{\omega_{p0}}\hat{E}_z - \frac{16}{3}a_1a_2\sin\theta = 0, \tag{5c}$$

where the intial conditions are given by $\hat{E}_z = 0$ and $\theta = 0$. Notice from Eq. (5a) that saturation occurs when $d\hat{E}_z/dt = 0$ at $\theta = \pi/2$.

In the absence of a frequency mismatch, $\Delta\omega_0 = 0$, θ increases monotonically from 0 to $\pi/2$, as indicated by Eq. (5b). Saturation occurs at $\theta = \pi/2$ and Eq. (5c) gives the value of the saturated electric field as $\hat{E}_{sat} = (16a_1a_2/3)^{1/3}$. However, analysis[10] of Eq. (5c) for $\theta = \pi/2$ indicates that the maximum saturation field occurs for the optimum frequency mismatch $\Delta\omega_0^{opt} = -(9a_1a_2/8)^{2/3}\omega_{p0}/2$ at which point $\hat{E}_{sat}^{max} = 4(a_1a_2/3)^{1/3}$. Also notice, that when $\Delta\omega_0 = \Delta\omega_0^{opt}$, Eq. (5b) indicates θ initially decreases until it reaches $-\pi/2$. At this minimum, $\theta = -\pi/2$, Eq. (5c) gives $\hat{E}_z = \hat{E}_{sat}^{max}/2$. The fact that θ initially decreases to the point $\theta = -\pi/2$, for the case where $\Delta\omega_0 = \Delta\omega_0^{opt}$, is important in the following discussions.

Qualitatively, the diffractive properties of the radiation beams in the PBWA may be examined by considering the effective index of refraction for each beam. This is done by a method identical to that used previously in the discussion of relativistic optical guiding.[4]

except that now the source current in the wave equation includes the density oscillations of the plasma wave, $J_\perp = -eca_\perp(n_0 + \delta n)$. The wave equation is given by

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) a_\perp = \frac{\omega_{p0}^2}{c^2} a_\perp \left[1 + \frac{\delta \hat{n}}{n_0}\sin(\Delta\phi + \theta)\right], \tag{6}$$

where $a_\perp = a_1 + a_2$. In short, the 1D limit of the Eq. (6) is taken assuming $a \sim \exp(i\phi)$ and then Eq. (6) is divided by the phase factor $\exp(i\phi)$ of either beam 1 or beam 2. The effective index of refraction is then obtained by averaging over a period of the radiation beat phase $\Delta\phi$. Neglecting relativistic effects, the effective index of refraction for each beam is given by

$$\eta_1 = 1 - \frac{\omega_{p0}^2}{2\omega_1}\frac{|\delta\hat{n}|}{2n_0}\frac{|a_2|}{|a_1|}(\sin\theta - i\cos\theta), \tag{7a}$$

$$\eta_2 = 1 - \frac{\omega_{p0}^2}{2\omega_2}\frac{|\delta\hat{n}|}{2n_0}\frac{|a_1|}{|a_2|}(\sin\theta + i\cos\theta). \tag{7b}$$

In the above expressions, the first term on the right (the unity) represents vacuum diffraction whereas the terms proportional to $|\delta\hat{n}|$ represent the diffractive properties of the plasma wave. The imaginary terms in the above expressions indicate power is being exchanged between the radiation beams and the plasma wave. (Notice that near $\theta \simeq 0$, Eqs. (7a) and (7b) indicate power is being transferred out of the higher frequency field and into the lower frequency field, as is the case for a Raman scattering process.)

First of all, notice that in deriving the above expressions for $\eta_{1,2}$, averages were taken over a period of the beat phase. This effectively removes any information on the length scale of the plasma wavelength and, hence, only the global properties of the radiation envelopes are described by the above expressions. Any behavior occurring on the plasma wavelength scale, such as the tendency for the radiation beams to break up into optically guided beamlets, has been removed by such an averaging. Secondly, notice that the focusing effects of the plasma wave on the global behavior of the radiation beams is strongly determined by θ, i.e., the shift in phase of the plasma wave away from the phase of the radiation beat wave. For example, near saturation, $\theta = \pi/2$, the plasma wave tends to defocus the radiation beams, assuming $|\delta\hat{n}| \sim \exp(-r^2/r_p^2)$ and $|a_1|/|a_2| \simeq 1$. On the other hand, $\theta = -\pi/2$ implies that the plasma wave leads to enhanced focusing of the radiation beams. (Recall from the above discussion that $\theta = -\pi/2$ when $E_z = E_{sat}^{max}/2$ for the case $\Delta\omega_0 = \Delta\omega_0^{opt}$.) In general, both the amplitude $|\delta\hat{n}|$ and phase shift θ will be functions of radius r (for example, plasma waves on axis may tend to saturate in a shorter distance than those at the edge of the radiation beams) and determining the diffractive effects of the plasma wave is not so straightforward. The above model illustrates the importance of the global behavior of the radiation envelopes on the phase shift θ, however, determination of the quantitative behavior of the radiation envelopes requires more detailed calculations.

Numerical studies of the focusing effects of the plasma wave in the PBWA have been performed by Gibbon and Bell.[7] In their analysis relativistic focusing effects have been neglected and any enhancement in the diffractive properties of the radiation beams is due solely to the density response of the plasma wave. In determining the density response of the plasma wave, however, the dynamics of energy exchange between the plasma wave and the radiation beams, including cascading to modes of frequency $\omega_l = \omega_1 \pm l\omega_p$ (where l is an integer), are calculated self-consistently. To study the focusing properties of the radiation beams, Gibbon and Bell derived envelope equations for each radiation mode of frequency ω_l. In deriving these envelope equations, however, averages where taken over the beat frequency $\Delta\omega \simeq \omega_p$ and, hence, any structure in the radiation envelopes occurring on a length scale λ_p is effectively removed (as is discussed above). Gibbon and Bell found that the focusing effects of the plasma wave on the radiation beam envelope depend strongly on the initial frequency mismatch $\Delta\omega_0$ between the radiation beat frequency and the ambient plasma frequency. When $\Delta\omega_0 = 0$, they found that resonant coupling to the plasma wave caused the radiation beam envelope to diffract more rapidly than it would in vacuum. For small negative values of $\Delta\omega_0$ (which corresponds to larger values for the saturation amplitude of the plasma wave), however, they found that resonant coupling to the plasma wave may lead to enhanced focusing of the radiation beam envelope.

This strong dependence of the focusing properties of the radiation on the frequency mismatch $\Delta\omega_0$, as observed by Gibbon and Bell,[7] may be physically understood by considering the small scale structure of the radiation beam envelope. As discussed above, a large amplitude plasma wave tends to break up the radiation beam into beamlets of length $\simeq \lambda_p/2$. When $\Delta\omega_0 = 0$, the plasma wave quickly reaches its saturation amplitude (a relatively short distance behind the radiation beam head) and the plasma wave is then 90° out of phase with the radiation beat wave. At the saturation point where $\theta = \pi/2$, the plasma wave is phased such that it leads to enhanced diffraction of the radiation beat wave. That is, the peaks of increased density in the plasma wave density variation coincide with peaks of the radiation $|a_1 + a_2|$ envelope, thus causing the radiation $|a_1 + a_2|$ envelope peaks to diffract more rapidly. This is illustrated schematically in Fig. 2.

For small negative values of $\Delta\omega_0$, however, saturation of the plasma wave is prolonged to a point further behind the radiation beam head and, initially, θ decreases (becomes negative). For the case $\Delta\omega_0 = \Delta\omega_0^{opt}$, then $\theta = -\pi/2$ when $E_z = E_{sat}^{max}/2$. At the point where $\theta = -\pi/2$, the peaks of decreased density in the plasma wave density variation coincide with the peaks of the radiation $|a_1 + a_2|$ envelope, thus causing enhanced focusing of the radiation $|a_1 + a_2|$ envelope peaks. This is illustrated schematically in Fig. 3. Hence, appropriate choices for the frequency mismatch $\Delta\omega_0$ may lead to optically guided beamlet regions which correspond to the peaks of the radiation $|a_1 + a_2|$ envelope. The envelope equations used in the analysis of Gibbon and Bell,[7] however, are averaged over a plasma period and, hence, only the global properties of the radiation envelope may be described. Any small scale structure, such as the formation of beamlets of length $\lesssim \lambda_p$, may not be described by the model of Gibbon and Bell.

IV. Conclusions

The above analysis indicates that when the plasma wave in the PBWA grows to a sufficiently large amplitude such that $|\delta n/n_0|$ satisfies the inequality expressed in Eq. (3), the plasma wave leads to periodic enhanced focusing of the radiation beams. In the absence of relativistic focusing effects the plasma wave tends to break up the radiation beams into axial beamlets of length $\lesssim \lambda_p/2$ which remain optically guided. These optically guided beamlets occur around the minima in the density oscillation of the plasma wave. (The sections of the radiations beams located in regions of increased density experience enhanced diffraction.) For the case of no initial frequency mismatch, $\Delta\omega_0 = 0$, the plasma wave quickly saturates at which point the plasma wave is 90° out of phase ($\theta = \pi/2$) with the radiation beat wave. In this case the regions about the peaks of the radiation $(a_1 + a_2)$ envelope experience enhanced diffraction. However, for an optimal choice of initial frequency mismatch, $\Delta\omega_0 = \Delta\omega_0^{opt}$, then θ becomes negative as the plasma wave electric field E_z increases. When $E_z = E_{sat}^{max}/2$, then $\theta = -\pi/2$. At this point the minima in plasma wave density coincide with the maxima in the magnitude of the radiation $|a_1 + a_2|$ envelope. Hence, when $\theta = -\pi/2$ it is possible for the peaks in the radiation $|a_1 + a_2|$ envelope to propagate as optically guided beamlets. This is important, since it is the ponderomotive force from the peaks in the envelope of the combined field $(a_1 + a_2)^2$ which drives the plasma wave. In such a way the resonantly generated plasma wave may lead to enhanced focusing of the radiation beams. A more thorough analysis of the focusing properties of the radiation in the PBWA requires the development of a self consistent model which includes the combined effects of relativistic optical guiding along with the effects of the resonantly generated plasma wave. This model must also be capable of describing the evolution of any fine scale envelope structure, such as the formation of radiation beamlets of size $\sim \lambda_p/2$.

Acknowledgements

The authors would like to acknowledge useful discussions with P. Gibbon and W.B. Mori. This work was supported by the U.S. Department of Energy and the Office of Naval Research.

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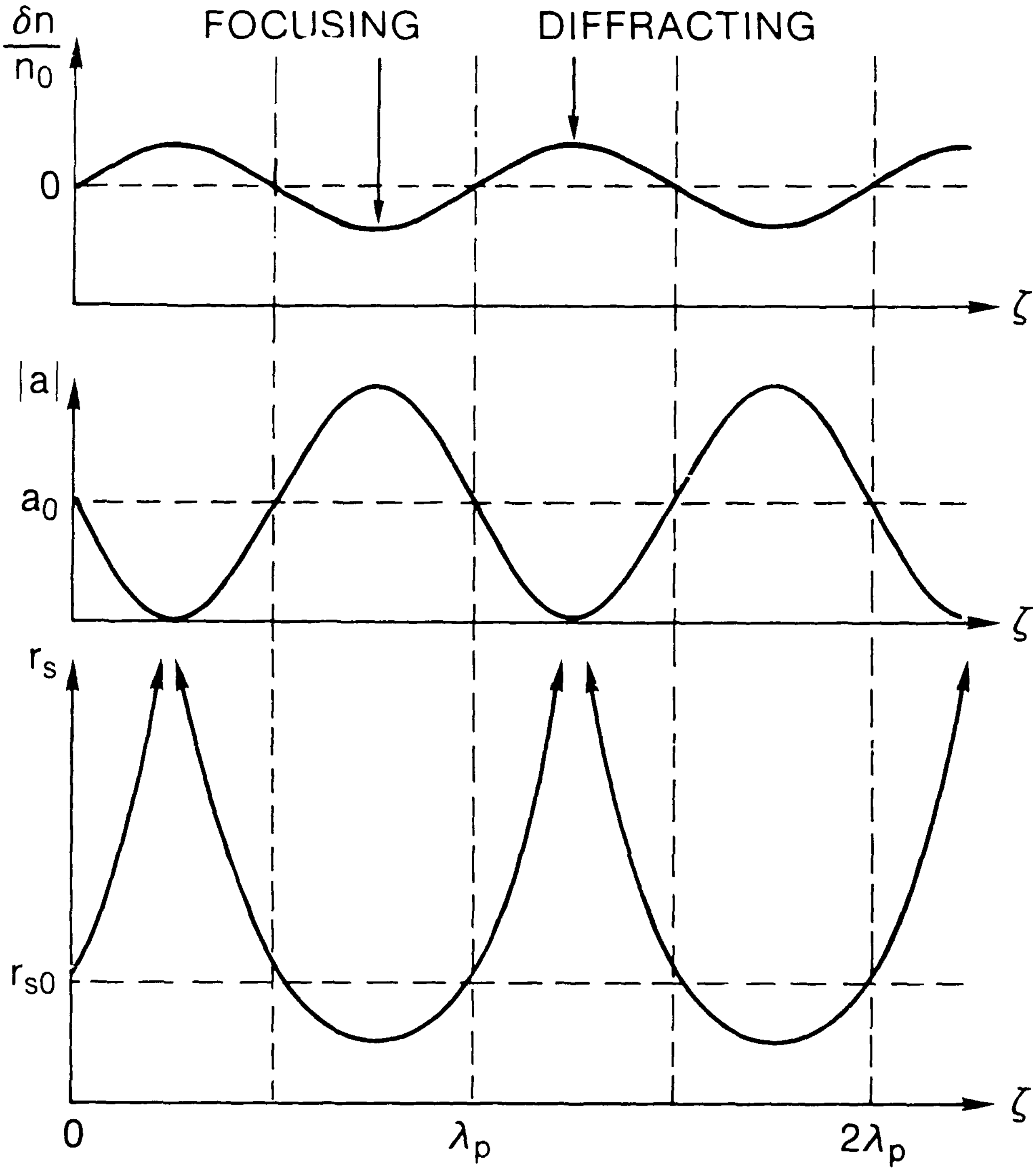

FIG. 1. Schematic illustrating the effects of an externally generated plasma wave on an initially uniform radiation beam. Here the radiation beam is shown to form periodic beamlet segments. Radiation in the regions center about the peaks of decreased electron density are optically guided, whereas radiation in the regions of increased density continually diffract.

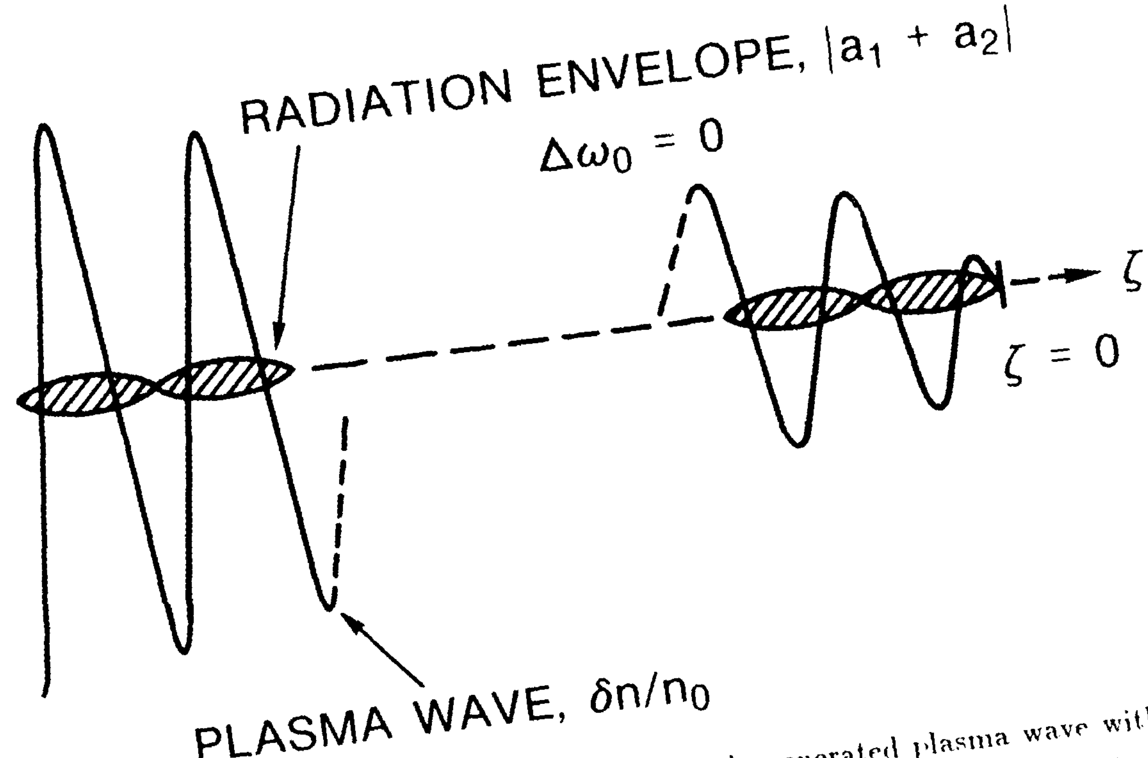

FIG. 2. Schematic of the interaction of the resonantly generated plasma wave with the radiation beat wave for the case of no initial frequency mismatch, $\Delta\omega_0 = 0$. Here the plasma wave saturates relatively quickly, at which point $\theta = \pi/2$. At saturation, the peaks in the radiation $|a_1 + a_2|$ envelope coincide with increases in the electron density, hence, the radiation envelope peaks will continually diffract.

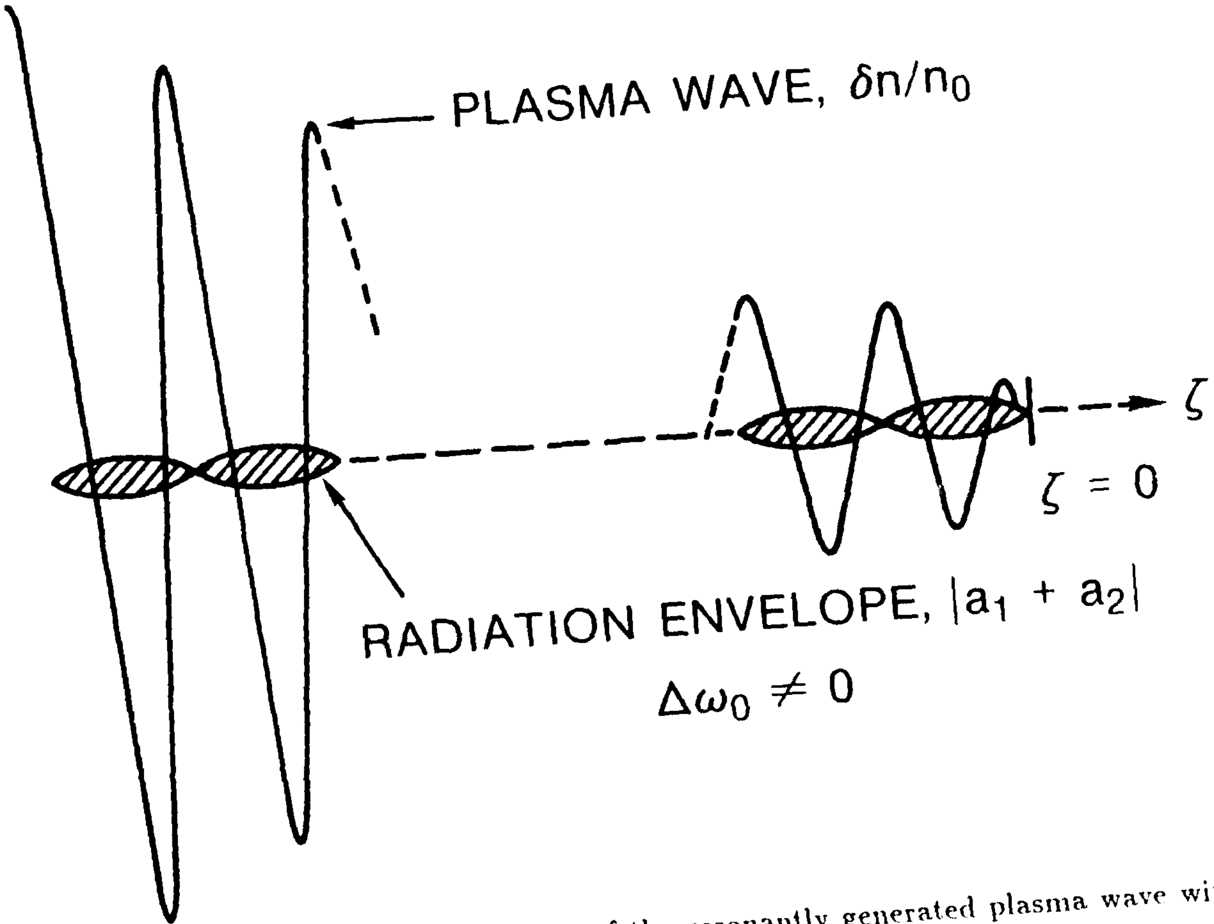

FIG. 3. Schematic of the interaction of the resonantly generated plasma wave with the radiation beat wave for the case of optimal initial frequency mismatch, $\Delta\omega_0 = \Delta\omega_0^{opt}$. For this case $\theta = -\pi/2$ when $E_z = E_{sat}^{max}/2$. Provided $\theta = -\pi/2$, then the peaks in the radiation $|a_1 + a_2|$ envelope coincide with the minima in the electron density, hence, the radiation envelope peaks will remain optically guided.

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